

Instanton induced quark dynamics and the pentaquark

N.I.Kochelev^{a,b*}, H.-J. Lee^{a†}, V. Vento^{a ‡}

(a) *Departament de Física Teòrica and Institut de Física Corpuscular,
Universitat de València-CSIC, E-46100 Burjassot (Valencia), Spain*

(b) *Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research, Dubna, Moscow region, 141980 Russia*

Abstract

We analyze the existence of the exotic Θ^+ from the perspective of instanton induced quark dynamics. The 't Hooft interaction gives strong attraction in specific channels of the triquark $ud\bar{s}$ and diquark ud configurations. In particular it leads to a light $ud\bar{s}$ triquark cluster, with the mass around 750 MeV, in the $I = 0$, $S = 1/2$ and color 3 configuration, and a light ud -diquark configuration, with mass 440 MeV, in the $I = 0$, $S = 0$ and color $\bar{3}$ configuration. If we consider the pentaquark as a bound state of such triquark and diquark configurations in a relative $L = 1$ state we obtain good agreement with the data. The small width of Θ^+ has a natural explanation in this model.

Pacs: 12.38.Aw, 12.38.Lg, 12.39.Ba, 12.39.-x

Keywords: quarks, instanton, hadrons, pentaquark

*kochelev@thsun1.jinr.ru

†Heejung.Lee@uv.es

‡Vicente.Vento@uv.es

1 Introduction

The discovery of the exotic Θ^+ baryon [1, 2, 3, 4], followed by recent evidences of narrow pentaquark states with strangeness $S = -2$ and charm [5, 6] has opened up a new scenario to understand quark dynamics, in particular at low energies where non perturbative mechanisms are expected.

There is a long history of predictions. The existence of exotics, with the quark content $udud\bar{s}$, have been proposed in the context of quark and bag models [7, 8]. However these states had large masses and typical hadronic widths. The soliton model of baryons, based on the implementation of spontaneous chiral symmetry breaking, was used for a successful prediction of a very narrow exotic pentaquark state, the Θ^+ , with the correct mass [9, 10, 11], despite the fact that recently some objections have been put forward to this analysis [12].

After the detection of the pentaquark a plethora of calculations have appeared aiming to understand the implications of its existence in low energy quark dynamics. Models to describe the complicated five particle scenario have been proposed [13, 14, 15, 16, 17]. In particular some models, like the ones proposed by Jaffe and Wilzek (JW), Karliner and Lipkin (KL), and Shuryak and Zahed (SZ), consider that colored quark clusters inside the pentaquark are formed and this explains its small mass and width. Their approach leads to appealing simplified dynamical schemes, but the need for quark clustering requires justification. The aim of this letter is to prove that the quark dynamics derivable from the instanton induced interaction justifies a certain type of clustering. Also various lattice QCD calculations have been carried out with contradictory results [18].

The instantons, strong fluctuations of gluon fields in the vacuum, play a crucial role in the realization of spontaneous chiral symmetry breaking in Quantum Chromodynamics and in the effective description of the spectroscopy for conventional hadrons. The instantons induce the 't Hooft interaction between the quarks which has strong flavor and spin dependence, a behavior which explains many features observed in the hadron spectrum and in hadronic reactions (see reviews [19, 20, 21] and references therein).

Particularly relevant for us here is that, in the quark-quark sector, the instanton induced interaction produces a strong attraction in flavor antisymmetric states. The strength of this interaction for the (ud) scalar diquark state is equal, for two colors, to the strength in the pion channel, the so-called Pauli-Gürsey symmetry, and only one-half weaker in the realistic $N_c = 3$ case [15]. As a result of this dynamics a quasi-bound very light ud -state can be formed. This mechanism implies that models for the pentaquark with diquark correlations are preferable to those without any correlation between the quarks. Furthermore the instanton induced interaction governs the dynamics between quarks at intermediate distances, i.e. $r \approx \rho_c \approx 0.3$ fm, where ρ_c is the average instanton size in the QCD vacuum [22]. This scale is much smaller than the confinement size $R \approx 1$ fm and therefore it favors the existence of clusters inside the large confinement region.

In this letter we consider a version of a triquark-diquark model for the pentaquark motivated by the instanton induced interaction between the quarks. We will show that taking into account the strong instanton interaction in triquark and diquark clusters allows us to understand the mass and width of the pentaquark.

2 Perturbative and non-perturbative interactions between quarks in multi-quark hadrons

Jaffe's famous papers on multi-quark states [23], based on the MIT bag model, motivated a wide discussion on the properties of exotic hadronic states. Most predictions have been based on the assumption that the perturbative one-gluon exchange interaction among quarks is the main mechanism to understand the spectrum of multi-quark systems. In an alternative approach [24, 25, 26], the non-perturbative instanton induced interaction has been suggested to dominate the spin and flavor dependent mass splitting between multi-quark states and to provide a very strong mixing between them.

The most important instanton induced interaction in quark systems is the multi-quark 't Hooft interaction, which arises from the quark zero modes in the instanton field (see Fig. 1).

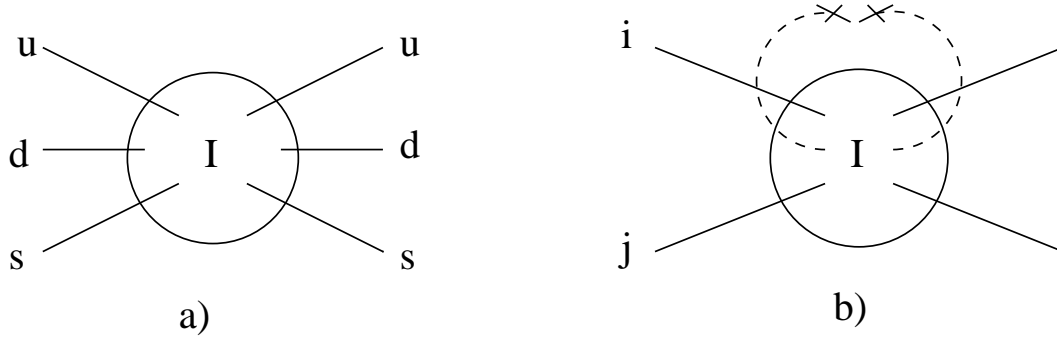


Figure 1: The instanton induced a) three-quark uds interaction and b) two-quark ud , us , ds interactions. In the figure I denotes the instanton, $i, j = u, d, s, i \neq j$.

For $N_f = 3$ (Fig. 1a) and $N_c = 3$ this interaction is given by [27]:

$$\begin{aligned} \mathcal{L}_{eff}^{(3)} = & \int d\rho \, n(\rho) \left\{ \prod_{i=u,d,s} \left(m_i^{cur} \rho - \frac{4\pi}{3} \rho^3 \bar{q}_i R q_{iL} \right) \right. \\ & + \frac{3}{32} \left(\frac{4}{3} \pi^2 \rho^3 \right)^2 \left[\left(j_u^a j_d^a - \frac{3}{4} j_{u\mu\nu}^a j_{d\mu\nu}^a \right) \left(m_s^{cur} \rho - \frac{4}{3} \pi^2 \rho^3 \bar{q}_{SR} q_{sL} \right) \right. \\ & + \frac{9}{40} \left(\frac{4}{3} \pi^2 \rho^3 \right)^2 d^{abc} j_{u\mu\nu}^a j_{d\mu\nu}^b j_s^c + \text{perm.} \left. \right] + \frac{9}{320} \left(\frac{4}{3} \pi^2 \rho^3 \right)^3 d^{abc} j_u^a j_d^b j_s^c \\ & \left. + \frac{i f^{abc}}{256} \left(\frac{4}{3} \pi^2 \rho^3 \right)^3 j_{u\mu\nu}^a j_{d\nu\lambda}^b j_{s\lambda\mu}^c + (R \longleftrightarrow L) \right\}, \end{aligned} \quad (1)$$

where, m_i^{cur} is the quark current mass, $q_{R,L} = (1 \pm \gamma_5) q(x)/2$, $j_i^a = \bar{q}_{iR} \lambda^a q_{iL}$, $j_{i\mu\nu}^a = \bar{q}_{iR} \sigma_{\mu\nu} \lambda^a q_{iL}$, ρ is the instanton size and $n(\rho)$ is the density of instantons⁴.

One can obtain an effective two-quark interaction induced by instantons from the three-quark interaction (1) by connecting two quark legs through the quark condensate (Fig. 1b). In the limit of small instanton size one obtains simpler formulas for effective

⁴For quarks with nonzero virtualities k_i^2 the vertex (1) should be multiplied by factors $Z_i = F(k_i^2)$ for each incoming and outgoing quark legs. For small values of the virtualities one can use the formula $F(k_i^2) \approx e^{-\rho\sqrt{k_i^2}}$.

two- and three-body point-like interactions [24, 25, 26]:

$$\begin{aligned}\mathcal{H}_{eff}^{(2)}(r) = & -V_2 \sum_{i \neq j} \frac{1}{m_i m_j} \bar{q}_{iR}(r) q_{iL}(r) \bar{q}_{jR}(r) q_{jL}(r) \left[1 + \frac{3}{32} (\lambda_u^a \lambda_d^a + \text{perm.}) \right. \\ & \left. + \frac{9}{32} (\vec{\sigma}_u \cdot \vec{\sigma}_d \lambda_u^a \lambda_d^a + \text{perm.}) \right] + (R \longleftrightarrow L),\end{aligned}\quad (2)$$

and

$$\begin{aligned}\mathcal{H}_{eff}^{(3)}(r) = & -V_3 \prod_{i=u,d,s} \bar{q}_{iR}(r) q_{iL}(r) \left[1 + \frac{3}{32} (\lambda_u^a \lambda_d^a + \text{perm.}) \right. \\ & + \frac{9}{32} (\vec{\sigma}_u \cdot \vec{\sigma}_d \lambda_u^a \lambda_d^a + \text{perm.}) - \frac{9}{320} d^{abc} \lambda^a \lambda^b \lambda^c (1 - 3(\vec{\sigma}_u \cdot \vec{\sigma}_d + \text{perm.})) \\ & \left. - \frac{9 f^{abc}}{64} \lambda^a \lambda^b \lambda^c (\vec{\sigma}_u \times \vec{\sigma}_d) \cdot \vec{\sigma}_s \right] + (R \longleftrightarrow L),\end{aligned}\quad (3)$$

where $m_i = m_i^{cur} + m^*$ is the effective quark mass in the instanton liquid. These forms are suitable for calculating the instanton induced contributions within a constituent quark picture.

In our estimates below, to avoid uncertainties in the parameters of instanton model (see recent discussion in [28]) and uncertainties in the shape of quark wave functions, associated with the confinement potential, we will treat the product of the strength of four-quark instanton interaction V_2 and the overlapping radial structures of the wave functions of the quarks (2) as a free parameter, as suggested some time ago by Shuryak and Rosner [29]. We will fix the value of this parameter by fitting the masses of the hadronic ground states: the baryon octet and decuplet, and the vector meson nonet. Thus, our two body instanton interaction will have the form

$$V_{inst}^{qq} = - \sum_{i \neq j} \frac{a}{m_i m_j} \left[1 + \frac{3}{32} (\lambda_u^a \lambda_d^a + \text{perm.}) + \frac{9}{32} (\vec{\sigma}_u \cdot \vec{\sigma}_d \lambda_u^a \lambda_d^a + \text{perm.}) \right], \quad (4)$$

for the quark-quark interaction. The color-spin structure of the instanton induced quark-antiquark interaction can be obtained from Eq. (2) by crossing

$$V_{inst}^{q\bar{q}} = - \sum_{i \neq j} \frac{a}{m_i m_j} \left[1 - \frac{3}{32} (\lambda_u^a \lambda_s^a + \text{perm.}) + \frac{9}{32} (\vec{\sigma}_u \cdot \vec{\sigma}_s \lambda_u^a \lambda_s^a + \text{perm.}) \right], \quad (5)$$

where

$$\lambda_{\bar{q}} = -\lambda^*, \quad \sigma_{\bar{q}} = -\sigma^* \quad (6)$$

are color and spin generators for the antiquark representation.

In addition to the instanton interaction, we will also include in the fit the perturbative one-gluon hyperfine interaction

$$V_{OGE}^{qq} = - \sum_{i > j} \frac{b}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \lambda_i^a \lambda_j^a, \quad (7)$$

between quarks. For the quark-antiquark OGE interaction one should use the substitution in Eq. (6) [23].

It is easy to show that three body instanton interaction does not contribute to the masses of the ground state baryons. Therefore its strength can not be fixed from such fit and should be estimated within some model. The estimate of the three-body instanton

induced contribution to the mass of multi-quark system was considered for the first time in [24], where its contribution to the mass of the H-dibaryon has been analyzed. This estimate was based on Shuryak's version of the instanton liquid model [22], in which the density of instantons has the form $n(\rho) \propto \delta(\rho - \rho_c)$. In this model one can obtain a relation between strengths of the three- and two-body instanton induced interactions for zero quark virtualities (see [24]).

$$V_3 = -V_2 \frac{4\pi^2 \rho_c^2}{3m_u m_d m_s} \quad (8)$$

We will use this relation below to estimate three-body contribution to pentaquark mass.

3 Masses of ground state hadrons

We use the following mass formula for the ground hadronic states

$$M_h = E_0^{B,M} + \sum_i N_i m_i + E_{I2} + E_{OGE}, \quad (9)$$

where N_i is number of the quarks with flavor i in the state. In Eq. (9)

$$\begin{aligned} E_{I2} &= \langle h | V_{I2} | h \rangle = - \sum_{i \neq j} \frac{a}{m_i m_j} M_{i,j}^{I2} \\ E_{OGE} &= \langle h | V_{OGE} | h \rangle = - \sum_{i > j} \frac{b}{m_i m_j} M_{i,j}^{OGE} \end{aligned} \quad (10)$$

are the matrix elements of the two-body instanton and OGE interactions. In comparison with the Shuryak and Rosner constituent quark model with two-body instanton induced interaction [29], we have added the OGE contribution and the parameter $E_0^{M,B}$. This new parameter represents the contributions from the confinement forces and breaks the additivity of the simple constituent quark model. For example, in the MIT bag model approach this term would arise as a consequence of the bag energy. We will assume that the value of this parameter is the same for all hadrons with equal number of the valence quarks. The values for the color-spin matrix elements for the hadronic ground states are shown in Table 1. We assume $m_u = m_d = m_0$ and neglect the mixing between the pseudoscalar octet and singlet meson states. In the vector meson nonet the ideal mixing for their wave functions has been assumed.

The result of our fit to the baryon and vector meson masses is shown in Table 2. The values for the parameters are

$$\begin{aligned} m_0 &= 263 \text{ MeV}, \quad m_s = 407 \text{ MeV}, \quad E_0^M = 214 \text{ MeV}, \\ E_0^B &= 429 \text{ MeV}, \quad a = 0.0039 \text{ GeV}^3, \quad b = 0.00025 \text{ GeV}^3. \end{aligned} \quad (11)$$

From Table 2 one can conclude that the one-gluon exchange interaction contributes little to the hadron masses ⁵ and the main contribution to the spin-spin splitting between hadron multiplets comes from the instanton induced interaction. This conclusion is in agreement with the results of the constituent quark model calculations with instanton forces using various forms of quark wave functions [31, 32, 33]. This result was also

⁵The value of strength of one-gluon exchange in Eq. (11) corresponds to $\alpha_s \approx 0.4$, if one uses MIT bag model quark wave functions with a bag radius $R \approx 1\text{fm}$.

<i>Hadron</i>	M_{00}^{OGE}	M_{0s}^{OGE}	M_{ss}^{OGE}	$M_{00}^{I^2}$	$M_{0s}^{I^2}$
π	16	0	0	6	0
K	0	16	0	0	6
η	16/3	0	32/3	-2	8
η'	32/3	0	16/3	-4	-8
ρ/ω	-16/3	0	0	0	0
K^*	0	-16/3	0	0	0
Φ	0	0	-16/3	0	0
N	8	0	0	9/2	0
Λ	16/3	8/3	0	3	3/2
Ξ	0	8	0	0	9/2
Σ	0	8	0	0	9/2
Δ	-8	0	0	0	0
Σ^*	0	-8/3	-16/3	0	0
Ξ^*	0	-8/3	-16/3	0	0
Ω	0	0	-8	0	0

Table 1: Two-body color-spin matrix elements of one-gluon and instanton interactions for ground state hadrons.

confirmed independently by the calculation of instanton effects on hadron masses within the QCD sum rule approach [34]. With the values of the parameters shown in (11), the masses of the pseudoscalar mesons are the following

$$\begin{aligned}
m_\pi &= 344 \text{ MeV (140 MeV)}, \quad m_K = 628 \text{ MeV (498 MeV)}, \\
m_\eta &= 709 \text{ MeV (550 MeV)}, \quad m_{\eta'} = 1302 \text{ MeV (960 MeV)},
\end{aligned} \tag{12}$$

where in parenthesis we wrote their experimental values.

One can see, that our model overestimates the masses of particles from the pseudoscalar nonet. This happens because we are neglecting the difference in size between the pseudoscalar and vector nonet mesons in our model. It is known that the one-gluon exchange contribution behaves as $\approx 1/R$ [30], while the instanton contribution as $\approx 1/R^3$ [31]. In the case of pseudoscalar octet both one-gluon and instanton exchanges give very strong attraction between the constituents, due to the large value of the color-spin matrix elements (see Table 1). This attraction leads to a small effective size for the quark-antiquark system. In fact, the size of the such systems should be comparable to the instanton size. The η' , on the other hand, results from a different behavior, the instantons give very strong repulsion in this channel and therefore the size of the η' must be larger than that of the vector meson⁶.

We parametrize the size dependence of the particles by a new parameter $r = R_{eff}/R$ which affects the instanton and one-gluon interactions. We take $R \approx 1\text{fm}$, the size of the conventional baryons. Thus, in (10) for the pseudoscalar mesons we multiply the one-gluon exchange contribution by $1/r$ and the instanton contribution by $1/r^3$. Moreover for those systems with a size comparable with the instanton size, one should introduce the

⁶A detailed discussion of the η' properties are beyond the scope of this article. We just mention that the instanton contribution leads to a large mass splitting between the η and η' . Therefore, the $U_A(1)$ problem, does not arise in the instanton model.

	ρ	ω	K^*	Φ	N	Λ
M_h^0	740	740	884	1028	1218	1362
E_{OGE}	19	19	12	8	-29	-25
E_{I2}	0	0	0	0	-254	-224
M_h	759	759	896	1036	935	1113
M_h^{exp}	770	780	896	1020	940	1116
	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
M_h^0	1362	1506	1218	1362	1506	1650
E_{OGE}	-19	-19	29	22	16	12
E_{I2}	-164	-164	0	0	0	0
M_h	1179	1323	1247	1384	1522	1662
M_h^{exp}	1192	1315	1236	1386	1532	1672

Table 2: The contributions to the baryon and vector meson nonet masses M_h that arise from the one-gluon exchange (E_{OGE}), the two-body instanton interaction (E_{I2}), the sum of quark masses and the confinement energy contribution. M_h^{exp} labels the data.

instanton form factor,

$$F(r) \approx e^{-2N\rho_c/R_{eff}}, \quad (13)$$

where $\approx 1/R_{eff}$ is the average quark virtuality in the system and $N = 2$ or $N = 3$ for the instanton induced two- and three-body interactions, respectively. The result of the new fit to the masses of the pseudoscalar octet gives

$$m_\pi = 112 \text{ MeV}, \quad m_K = 498 \text{ MeV}, \quad m_\eta = 581 \text{ MeV} \quad (14)$$

in a good agreement with the data. As a result of the fit we observe that the effective size, of systems with strong instanton attraction, becomes

$$R_{eff} \approx 0.72R, \quad (15)$$

where conventionally $R \approx 1\text{fm}$. Of course, this estimate is rather rough, but it shows that one can expect that the quark systems with strong instanton attraction are small compared to the others ⁷.

4 Diquark-triquark model for pentaquark and instantons

In the previous section the strong influence of the instantons on the dynamics of the colorless quark systems has been shown. Here the application of the model to the color quark systems with instanton attraction within the five quark pentaquark system will be considered.

Let us start with the discussion of the $\Theta^+ udud\bar{s}$ wave function in the model with instanton induced correlations between quarks. The observed Θ^+ state is very light in comparison with the expectation of the constituent quark model for the typical mass of

⁷In the bag model the size of the system is determined by the position of minimum of the hadronic mass as a function of bag radius R and a strong instanton interaction shifts downward the position of the minimum.

the $udud\bar{s}$ system⁸, and has a very small width. Thus, we should expect a non trivial wave function for the pentaquark. One of the peculiarities of the instanton induced interaction is its strong flavor dependence, i.e., it is not vanishing only for the interaction among quarks of different flavor. For the ud diquark system the strong instanton attraction is possible only in isospin $I = 0$ channel. Thus, preferably the configuration in the $udud$ subsystem will be two separated isoscalar ud diquarks. The remaining antiquark \bar{s} can join one of the diquarks to create a triquark $ud\bar{s}$ configuration in the instanton field. In this triquark state all quarks have different flavors, therefore the instanton interaction is expected to be maximal. Another peculiarity of the instanton interaction is that it is maximal in the system with the minimal spin. Therefore, a pentaquark configuration with $S = 1/2$ $ud\bar{s}$ triquark and ud $S = 0$ diquark should be preferable. Therefore our final triquark-diquark picture for the pentaquark with instanton forces between quarks arises as shown in Fig.2a, where the triquark is a quasi-bound state in the field of the instanton (antiinstanton) and the diquark is a quasi-bound state in the antiinstanton (instanton) field⁹. To avoid the coalescence of the triquark-diquark state into the single cluster $udud\bar{s}$ configuration, where the instanton interaction is expected to be much weaker, due to the Pauli principle for the same flavor quarks in instanton field, we assume a non-zero orbital momentum $L = 1$ in the triquark-diquark system. The centrifugal barrier protects the clusters from getting close and prohibits the formation of the much less bound five quark cluster.

It should be mentioned, that, from our point of view, the possibility of a pentaquark configuration formed by two ud -diquark clusters and a single antiquark \bar{s} , shown in (Fig. 2b), as implied by the JW and SZ models, is suppressed by extra powers of the instanton density, $f = n_{eff}\pi^2\rho_c^4 \approx 1/10$ in the instanton model as compared with the triquark-diquark configuration of Fig. 2a.

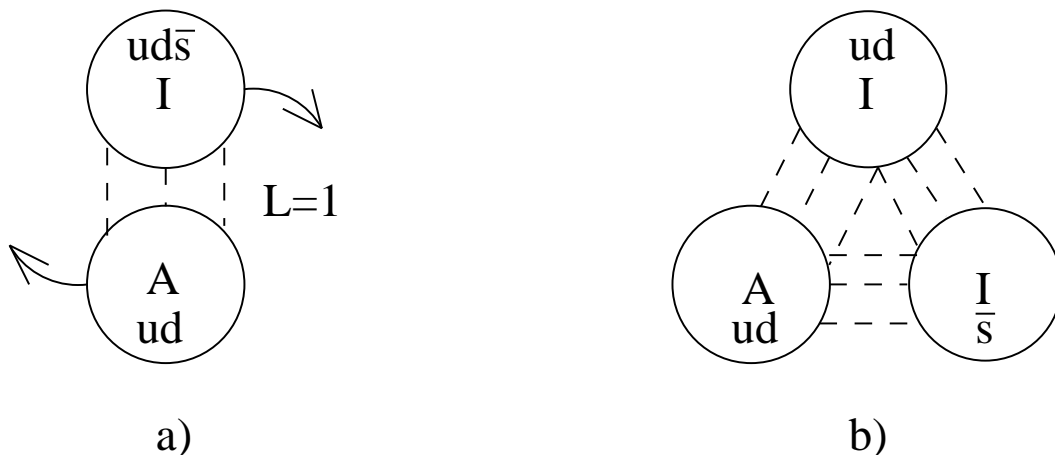


Figure 2: (a) Our instanton model for the pentaquark, (b) is the instanton picture for JW and SZ models. I (A) denote instanton (antiinstanton) configurations. Dashed lines indicate gluon lines.

⁸One can estimate it by using our fit above $M(udud\bar{s}) \approx E_0^B + E_0^M + 4m_0 + m_s \approx 2100$ MeV, which is much larger than experimental value $M_{\Theta^+} \approx 1540$ MeV.

⁹There is attraction (repulsion) between pseudoparticles with the same (opposite) topological charge. Therefore the instanton-antiinstanton (IA) configuration has smaller energy than the II and AA configurations.

According to Pauli statistics in the $ud\bar{s}$ $I = 0$ triquark state the ud diquark can be in $S = 0$ spin and $\bar{3}_c$ color state (A state) or in $S = 1$, 6_c (B state). In KL only B has been considered. In fact, there is a strong mixing between the two states due to both the one-gluon and instanton interactions (see below), and one cannot neglect either.¹⁰ In Table 3, the diagonal ($\langle A|H|A\rangle$, $\langle B|H|B\rangle$) and the non-diagonal ($\langle A|H|B\rangle$) color-spin matrix elements of the one-gluon and instanton interactions for the $ud\bar{s}$ triquark and scalar-isoscalar ud diquark are shown. For the instanton interaction we also have included the three-body color-spin M^{I3} matrix elements of the interaction (3). Its explicit form for the $ud\bar{s}$ state is

$$E_{ud\bar{s}}^{I3} = b_3 \left[1 + \frac{3}{32}(\lambda_u^a \lambda_d^a - \lambda_u^a \lambda_{\bar{s}}^a - \lambda_d^a \lambda_{\bar{s}}^a) + \frac{9}{32}(\vec{\sigma}_u \cdot \vec{\sigma}_d \lambda_u^a \lambda_d^a + \text{perm.}) \right. \\ \left. + \frac{9}{320} d^{abc} \lambda_u^a \lambda_d^b \lambda_{\bar{s}}^c (1 - 3(\vec{\sigma}_u \cdot \vec{\sigma}_d - \vec{\sigma}_u \cdot \vec{\sigma}_{\bar{s}} - \vec{\sigma}_d \cdot \vec{\sigma}_{\bar{s}})) \right. \\ \left. - \frac{9}{64} f^{abc} \lambda_u^a \lambda_d^b \lambda_{\bar{s}}^c (\vec{\sigma}_u \times \vec{\sigma}_d) \cdot \vec{\sigma}_{\bar{s}} \right], \quad (16)$$

State	M_{00}^{OGE}	M_{0s}^{OGE}	M_{00}^{I2}	M_{0s}^{I2}	M^{I3}
ud	8	0	3	0	0
$\langle A H A\rangle$	8	0	3	9/4	9/2
$\langle B H B\rangle$	4/3	40/3	3/2	51/8	27/4
$\langle A H B\rangle$	0	$-(96)^{1/2}$	0	$-(243/32)^{1/2}$	$-(243/8)^{1/2}$

Table 3: Color-spin matrix elements of the one-gluon and instanton interactions for triquark $ud\bar{s}$ and diquark ud states

From Table 3 it can be seen that the one-gluon and instanton two body interactions give strong attraction in the diquark and triquark channels. For example, comparison of the matrix elements in Table 1 and Table 3, in the $SU(3)_f$ symmetry limit, show that, in the B triquark state, the attraction is even larger than in the case of the Goldstone pion! Therefore, one can expect very light triquark cluster type configurations whose size is that of the pion. We see in Table 3 that the off-diagonal matrix elements between the states A and B are rather large and the physical states arise as a mixing of these states. Three-body instanton induced forces give a repulsion in both A and B states due to the opposite sign of their strength, (8). The final result depends on the overlap between the $ud\bar{s}$ quark wave functions. We estimated three-body contribution by using the bag model wave functions. The final result is

$$\Delta M_{I3} \approx -0.03 M^{I3} \Delta M_{I2}^N \frac{4\pi^2 \rho_c^2}{3m_s R^3 r^6} e^{4\rho_c/R(1-3/2r)}, \quad (17)$$

where ΔM_{I2}^N is the instanton contribution to the nucleon mass, $r = R_{tri}/R$ and we take into account the difference in form factors for the two- and three-body interactions, Eq. (13). We assume that due to the similarity in strength of the instanton attraction in the pseudoscalar octet, triquark and diquark configurations, the size of the all these states is the same. Thus we may use the same value for parameter r

$$r = R_{eff}/R \approx R_{tri}/r \approx R_{di}/R. \quad (18)$$

¹⁰Other types of mixing have also been discussed in the literature [17].

Finally we have

- diquark : $M_{di} = 442 \text{ MeV}$, $M_{0di} = 740 \text{ MeV}$,
 $\Delta M_{OGE} = -24 \text{ MeV}$, $\Delta M_{I2} = -274 \text{ MeV}$;
 - triquark A : $M_{tri} = 955 \text{ MeV}$, $M_{0tri} = 1362 \text{ MeV}$,
 $\Delta M_{OGE} = -40 \text{ MeV}$, $\Delta M_{I2} = -407 \text{ MeV}$, $\Delta M_{I3} = 40 \text{ MeV}$;
 - triquark B : $M_{tri} = 859 \text{ MeV}$, $M_{0tri} = 1362 \text{ MeV}$,
 $\Delta M_{OGE} = -50 \text{ MeV}$, $\Delta M_{I2} = -513 \text{ MeV}$, $\Delta M_{I3} = 60 \text{ MeV}$;
 - off – diagonal AB : $\Delta M_{OGE} = 32 \text{ MeV}$, $\Delta M_{I2} = 164 \text{ MeV}$,
 $\Delta M_{I3} = -49 \text{ MeV}$,
- (19)

where M_0 is the mass of the state without the one-gluon and instanton contributions. From (19) it follows that the two-body instanton interaction gives a very large and negative contribution to the masses for all diquark and triquark states. At the same time, the one-gluon contribution is rather small. After diagonalization of the mass matrix for the A and B states, we obtain for two mixed triquark states

$$M_{light}^{tri} = 753 \text{ MeV} \text{ and } M_{heavy}^{tri} = 1061 \text{ MeV}. \quad (20)$$

Comparing the masses of non-mixed (19) and mixed (20) states we see that the mixing is an important effect in the spectroscopy of the triquark states. It increases the difference between the two states from 96 MeV to 308 MeV, producing a very light $ud\bar{s}$ triquark state with a mass 753 MeV. It is about 360 MeV more bound than the lightest uds Λ state. The reason is simple. Both the one-gluon and instanton interactions are twice more attractive in the quark-antiquark channel than in the quark-quark case. The mass of light triquark cluster is smaller than the sum of masses of the K meson and the constituent u and d quarks. Therefore, the pentaquark cannot dissociate to the $Ku(d)$ system. Thus, the Θ^+ , as a system of light triquark and diquark clusters, can decay only by rearrangement of the quarks between these clusters. However, this rearrangement is highly suppressed by the orbital momentum $L = 1$ barrier between the clusters. As a consequence, the centrifugal barrier, provides the mechanism for a very small width in the case of the Θ^+ . The other, heavy triquark, 1061 MeV state (20), can easily dissociate to the $Ku(d)$ system. For this state, which should be approximately 300 MeV above of Θ^+ state, a very large width is expected. We have found also a rather small mass 442 MeV for the $S = 0$, $I = 0$ ud -diquark. This mass is in agreement with the estimate of $\approx 420 \text{ MeV}$ for this diquark obtained within the QCD sum rule approach using instanton induced interactions [35].

Finally let us estimate the total mass of Θ^+ if built as a system of a triquark cluster with mass 753 MeV, a diquark cluster with mass 442 MeV bound together in relative $L = 1$ orbital momentum state. The reduced mass for such triquark-diquark system is $M_{red}^{tri-di} = 279 \text{ MeV}$. This mass is approximately equal to the "effective" reduced mass of the strange quarks in the Φ meson, $M_{red}^{\Phi} \approx M_{\Phi}/4 = 255 \text{ MeV}$. For two strange quarks, the $L = 1$ energy of orbital excitation, can be estimated from the experimental mass shift between Φ meson and the $L = 1$ $f_1(1420)$ state

$$\Delta E(L = 1) \approx M_{f_1(1420)} - M_{\Phi} = 400 \text{ MeV}. \quad (21)$$

By neglecting the small difference between the reduced mass in strange-antistrange quark system and triquark-diquark system, we estimate the mass of the light pentaquark in our triquark-diquark cluster model with instanton interaction as

$$M_{\Theta^+} = M_{light}^{tri} + M_{di} + \Delta E(L = 1) \approx 1595 \text{ MeV}, \quad (22)$$

which is close to the data.

We should mention that our estimate of the $L = 1$ excitation energy is larger by a factor of two than the KL estimate (207 MeV) [14]. The KL estimate has been obtained from the assumption that due to the approximately equal reduced mass of the triquark-antidiquark and $c\bar{s}$ systems, this energy is equal to the $L = 1$ excitation energy in D_s mesons. An additional assumption was done in interpreting the new $D_s(2317)$ state [38] as a 0^+ excitation of $0^-D_s(1969)$. We don't want to discuss here the rather controversial status of the $D_s(2317)$ in the constituent quark model ¹¹, we would like to emphasize simply that the reduced mass for the triquark-diquark system of 458 MeV in their equation (2.4) was been obtained without the contribution from the hyperfine interaction. After including this effect one gets for the triquark state 890 MeV and 495 MeV for mass of the diquark state. As a result, the corrected reduced mass triquark-diquark system in KL model is 318 MeV which is much smaller then reduced mass of $c\bar{s}$ system 410 MeV in the KL paper. We can estimate the orbital excitation energy for the KL model by using simple dependence of this energy on the reduced mass of the system [36]¹²

$$\Delta E(L = 1) \propto M_{red}^{-n/(n+2)}, \quad (23)$$

which one obtains from the solution of the Schrödinger equation in a confinement potential $\propto r^n$. By putting conveniently $n = 1$ in (23) and using experimental information on the $L = 1$ excitation energy in the $s\bar{s}$ system, we estimate

$$\Delta E(L = 1)_{KL}^{corrected} \approx 370 \text{ MeV}. \quad (24)$$

This value is much larger then original KL estimate of 207 MeV and should lead to a significant deviation of their final result for the mass of Θ^+ from the experiment.

5 Conclusion

We have suggested a triquark-diquark model for the pentaquark based on instanton induced interaction. It is shown that this strong interaction, which is at the origin of the light pseudoscalar octet of the mesons, leads also to the very light $ud\bar{s}$ triquark and ud diquark color states. As the result, the possibility to explain the smallness of both, mass and width, for the observed Θ^+ based on triquark-diquark model with strong instanton attraction between quarks, has been shown.

Let us discuss another possible signals for existence of very light $ud\bar{s}$ triquark state. One interesting multiquark system with expected small width can be a triquark-antitriquark system with non-zero orbital momentum. The estimates of the mass of such system in $L = 1$ state within our model gives the number

$$M_{tri-antitri} = 2M_{light}^{tri} + \Delta E(L = 1, M_{red} = 377 \text{ MeV}) \approx 1860 \text{ MeV} \quad (25)$$

This mass is slightly smaller that two nucleon masses $2M_N = 1880 \text{ MeV}$ and therefore this triquark-antitriquark state can provide a new explanation of the large near threshold enhancement in $p\bar{p}$ spectrum in the reaction $J/\Psi \rightarrow \gamma p\bar{p}$ found by the BES Collaboration [37].

¹¹It is rather difficult to explain the small mass of this meson in the constituent quark model (see references in [38].)

¹²We are grateful Sergo Gerasimov for discussion the problems of estimating the energy of orbital excitations in the constituent quark model.

One also can consider a system consisting of a light ($u\bar{d}s$)-triquark and a flavor antisymmetric us or ds diquark. According of our model the us and ds diquarks should be heavier by 250 MeV and therefore its mass should be around 1800 MeV. We should mention, that in our model we do not expect very narrow multiquark states with $u\bar{d}s$ or $d\bar{u}s$ -triquark state clusters inside. The reason is simple. Due to small mass of the pion this triquark can easy dissociate to pion and a constituent strange quark ¹³.

We should emphasize, that due to the specific properties of the $u\bar{d}s$ light quark state, it should play the important role not only in the spectroscopy of the multiquark states, but also in different hadronic reactions. This triquark may also give the rise to properties of the quark-gluon plasma and nuclear matter.

Acknowledgments

We are grateful to A.E.Dorokhov and S.B.Gerasimov for useful comments. N. I. Kochelev is grateful to the University of Valencia for the warm hospitality and financial support. V. Vento thanks CERN-TH for the hospitality during the early stages of this work. H.-J. Lee is a Postdoctoral fellow from SEEU-SB2002-0009. This work was supported by grants MCyT-FIS2004-05616-C02-01 and GV-GRUPOS03/094 (VV), and RFBR-04-02-16445, RFBR-03-02-17291 and INTAS-00-00-366 (NIK).

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¹³The first evidence for the candidates for the narrow pentaquark states with quark content $d\bar{u}sds$ and $u\bar{d}sds$ was found by NA49 Collaboration [5]. These results have been criticized in [39].

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